Model Examination PaperI
MATHEMATICS
Class : XII

Time Allowed : 3 Hrs Maximum Marks: 100

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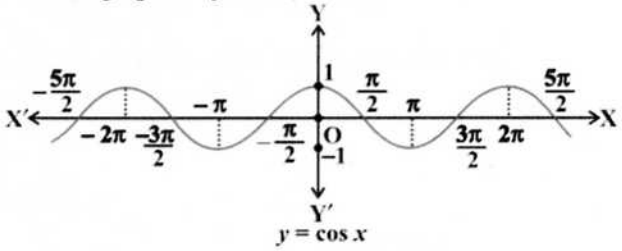
1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If $A = \{ a, b, c \}$ and $B = \{ 1, 2, 3 \}$ and a function $f: A \rightarrow B$ is given by $f = \{ (a, 2), (b, 3), (c, 1) \}$. Is f a bijective function?

2. In this graph of $y = \cos x$,write the intervals of x in which the function can be inverted.



3. If A and B are square matrices of the same order . Check whether
$$(A + B)^2 = A^2 + 2AB + B^2$$

4. If a matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$, Show that $(A - A')$ is a skew symmetric matrix , where A' is the transpose of matrix A.

5. If $A = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$ and $A^2 = \begin{pmatrix} -5 & -18 \\ 18 & 7 \end{pmatrix}$ find $A^2 - 6A + 17I$.

6. Find $\int \frac{1}{\sqrt{9-25x^2}} dx$

7. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

8. Find the components and magnitude of the vector $\overrightarrow{PQ}$, where P has coordinates (-1,-2,4) and Q has coordinates (2,0,-2).

9. Show that the points A(3,-5,1) , B (-1, 0 , 8) and C (7 , -10 , -6) are collinear.

10. Find $\int_{-\pi/2}^{\pi/2} \sin^7 x\,dx$

SECTION – B

11. Let A = Q x Q , Q being the set of rationals . Let '*' be a binary operation on A , defined by (a, b) * (c , d) = (ac , ad + b) . Show that

(i) '*' is not commutative (ii) '*' is associative

(iii The Identity element w.r.t '*' is (1 , 0)

OR

Let '*'be a binary operation on the set { 0,1,2,3,4,5} and

$$a*b = \begin{cases} a+b \text{ if } & a+b<6 \\ a+b-6 \text{ if } & a+b \geq 6 \end{cases}$$

Find the identity element and the inverse element of each element of the set for the operation '*'.

12. Solve the Equation

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x, (x > 0)$$

13. Is the given function continuous at x = 0?

$$f(x) = \begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

14.Find $\frac{dy}{dx}$, when $y = \sqrt{a+\sqrt{a+\sqrt{a+x^2}}}$, where a is a constant .

15. Discuss the applicability of Lagrange's mean value theorem for the function:

f(x) = |sinx| in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

16. Evaluate the integral $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

OR

Evaluate $\int_0^{\pi/2} \sin 2x \ \tan^{-1}(\sin x)dx$

17.Form the differential equation satisfied by $\sqrt{[1-x^2]} + \sqrt{[1-y^2]} = a.(x-y)$, a is an arbitrary constant.

18. Solve the differential equation $(x-1)dy + y\,dx = x(x-1)\,y^{1/3}\,dx$

OR

Solve the differential equation: $\sec^2 y\frac{dy}{dx}+2x\tan y = x^3$

19. Find the angle between $\vec{a}$ and $\vec{b}$. If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3, |\vec{b}|=5$ & $|\vec{c}|=7$

OR

Find λ if the vectors $\vec{a}=\hat{i}-\lambda\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}-5\hat{j}+2\hat{k}$ are perpendicular to each other.

20. Find the distance of the point (2, 4, -1) from the line
$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$

21. A die is tossed thrice. Find the probability of getting an odd number at least once.

22. Find the interval in which the value of the Determinant of the matrix A lies .

Given $A=\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

SECTION – C

23. Find A^{-1}, by using elementary row transformations . Given $A=\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$.

24. Show that the height of the cylinder of greatest volume that can be inscribed in a right circular cone of height h and having semi vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3\tan^2\alpha$

25. Prove that the curves $y = x^2$ and $x = y^2$ divide the square bounded by x = 0 , y=0 , x=1 and y=1 into three parts that are equal in area.

26. The chances of a patient having a heart attack is 40%. According to latest research Drug A reduces the risk of heart attack by 30% and drug B reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient had been prescribed Drug A.

OR

A factory manufactures screws, machines X, Y and Z manufacture respectively 1000,2000, 3000 of the screws,1%, 1.5% and 2 % of their outputs are respectively defective .A screw is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine X ?

27. Find the equation of two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$.

28.

Find $\int \frac{x^4\,dx}{(x-1)(x^2+1)}$

OR

Find $\int \left[\sqrt{\cot x}+\sqrt{\tan x}\right] dx$

29. Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can stitch 6 shirts and 4 pants] day while B can stitch 10 shirts and 4 pants per day . How many days shall each work , if it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost ? Solve the problem graphically.