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Model Examination Paper II
MATHEMATICS
Class : XII

Time Allowed : 3 Hrs | Maximum Marks: 100

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Find the Matrix X that it satisfies the equation A - 2 B + X = 0, given that

$$A=\begin{bmatrix}5 & 3\\ -3 & 1\end{bmatrix}; B=\begin{bmatrix}0 & -2\\ 3 & 1\end{bmatrix}$$

2.A and B are symmetric matrices , prove that AB - BA is a skew symmetric matrix .

3.Let A = {p, q , r } and B = {4, 5, 6}. If f is a function from A to B , such that f = { (p,6) , (q , 5) , (r , 4) } , find f^{-1}

4. Evaluate the integral $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$

5. Evaluate $\sin^{-1}(-\frac{\sqrt{3}}{2})$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6. Evaluate the determinant $\begin{vmatrix}\sin 10^\circ & -\cos 10^\circ\\ \sin 80^\circ & \cos 80^\circ\end{vmatrix}$

7.Find the value of $\int_0^1 xe^x dx$

8.If $\vec{a}, \vec{b}, \vec{c}$ are non zero vectors and $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$, show that $\vec{c}$ is perpendicular to $\vec{a}-\vec{b}$

9. If $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}-4\hat{j}+7\hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$

10. Two sides of a triangle are formed by the two vectors, $\vec{a}=3\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{b}=4\hat{i}-\hat{j}+7\hat{k}$. Find its third side

SECTION – B

11. Show that x = 2 is a root of the equation formed by the following determinant and hence solve the equation..

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

12. A random variable X has the following probability distribution , find

(i) The value of K (ii) $P(X \leq 1)$ (iii) $P(X > 3)$

X	0	1	2	3	4	5
P(X)	0.1	K	0.2	2K	0.3	K

13. Differentiate $\tan^{-1}\left(\frac{a\sin x+b\cos x}{a\cos x-b\sin x}\right)$, w . r .t x

OR

Find $\frac{dy}{dx}$, when $x=a\left[\cos t+\log\left(\tan\frac{t}{2}\right)\right]$; y= a sint

14. Evaluate $\int\frac{x}{\sqrt{8+x-x^2}}dx$

OR

Evaluate: $\int\frac{1}{3+\sin^2 x}dx$

15.If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$

16. $f: R \to A, A=\{x: x\in R, -1<x<1\}, f(x)=\frac{x}{1+|x|}, x\in R.$

Show that the function f is a bijective function.

17.Show that the vectors $\vec{a}=\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}-4\hat{k}, \vec{c}=3\hat{i}+2\hat{j}-1\hat{k}$ are linearly independent.

OR

Find a , if the points A(10,3) , B (12, -5) and C (a , 11) are collinear.

18. If $|\vec{a}|=5, |\vec{b}|=13, \vec{a}.\vec{b}=60$, find $|\vec{a}\times\vec{b}|$.

19. Form a differential equation corresponding to the function $y = c\,(x - c)^2$

OR

Form a differential equation corresponding to the function $y = a e^{x} + b e^{2x}$

20. Solve the differential equation $x^2dy + y(x+y)dx = 0$, when $x=1,\ y=1$.

21._ Verify Rolle's theorem for the function:

$$f(x) = x^2 - 5x + 4 \text{ on } [1,4]$$

22. Examine the continuity of the cosine function.

SECTION – C

23.Solve the following system of equations , using Matrices

$$4x + 2y + 3z = 2 ; x + y + z = 1;3x + y - 2z = 5$$

24. A catering agency has kitchen at two places A and B . From these places , supply is made to each the three schools situated at P, Q and R for mid-day meals . The monthly requirements of the the three schools are repectively 40, 40 and 50 food packets while the production capacity of the kitchens at A and B are 60 and 70 packets respectively. The transportation cost per packet from the kitchens to the schools are given below.

Transportation Cost per packet (in Rupees)		
To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to each school so that the cost of transportation is minimum? Also find the minimum cost.

25. Find the area of the region bounded by lines

$$y=\frac{5}{2}x-5; x+y-9=0; y=\frac{3}{4}x-\frac{3}{2}$$

OR

Find the area of the smaller region bounded by the curves $x^2 + y^2 = 4$ and $y^2 = 3(2x - 1)$.

26. Find the vector equation of the line parallel to the line $\frac{x-1}{5}=\frac{3-y}{2}=\frac{z+1}{4}$ and passing through the point(3, 0, – 4). Also find the distance between these two lines.

27 Two bags A and B contain 3 red 4 black balls , and 4 red and 5 black balls respectively. From bag A one ball is transferred to bag B and then a ball is drawn from bag B. The ball is found to be red in colour. Find the probability that

(a)the transferred ball is black ?

(b) the transferred ball is red ?

OR

Find the probability distribution of doublets in three throws of a pair of dice

28. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

29. Evaluate $\int_0^2 |x^2 + 2x - 3| dx$

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