Model III Examination Paper
MATHEMATICS
Class : XII

Time Allowed : 3 Hrs Maximum Marks: 100

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1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Form a 2 x 2 matrix $A = [a_{ij}]$ where a_{ij} is given by

$$a_{ij} = \frac{|2i - j|}{3j}$$

2. Differentiate $e^{x^2+\tan x}$, w.r.t x

3. Evaluate the integral $\int \frac{\left(\tan^{-1}(x+5)\right)^8}{1+x^2} dx$

4. In a single throw of two dice , find the probability of getting numbers whose product is six .

5. Ten eggs are drawn successively from a lot containg 10% defective eggs. Find the probability of atleast one defective egg.

6. Show that function $f(x) = \cos x$ is neither increasing nor decreasing on $(0, 2\pi)$.

7. Find whether $y = \frac{a}{x} + b$, is a solution of $\frac{d^2y}{d^2x} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

8. For what value of k, the matrix $\begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$ does not have an inverse.

9. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ find a and b.

10. For what values of 'λ', are the vectors $(2\hat{i} - 3\hat{j})$ and $(\lambda\hat{i} - 6\hat{j})$ parallel ?

SECTION – B

11. Find the derivative of $(\log x)^{\log x}, x > 1$

12. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces

13. For what value of k is the following function continous at x = 1

$$f(x)=\begin{cases}\dfrac{x^2-1}{x-1}, & x\neq 1\\ 4k, & x=1\end{cases}$$

14.In an experiment , p the probability of success is twice the probability q of its failure . Find the probability that in the next 6 trials there will be atleast 4 successes

15. Find the points on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$

16. Write into the simplest form:

$$\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

17. If a, b, c are in A.P. find the value of determinant

$$\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

OR

Let a , b , cbe positive numbers , but not all equal.

Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

18.If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$

OR

Solve the differential equation : $(x - 1)\dfrac{dy}{dx} = 2xy$, *If* $y(2)=1$.

19. Evaluate $\int (2\tan x+3\cot x)^2 dx$

20. Evaluate $\int \dfrac{1}{\sqrt{(x-a)(x-b)}}dx$

OR

Evaluate $\int \frac{x - \sin x}{1-\cos x} dx.$

21.Evaluate $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

22.ABCD is a parallelogram with $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}; \overrightarrow{AD} = \hat{i} - 2\hat{j} - 3\hat{k}.$

Find a unit vector parallel to its diagonal $\overrightarrow{AC}$.Also, find the area of the parallelogram ABCD

OR

Find the projection of $(\vec{b}+\vec{c})$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

SECTION – C

23.A dietician wishes to mix low calorie foods A and B in a way that, the foods contain atleast 40 units of vitamins , 50 units of minerals and 35 calories .
Two foods A and B are available at a cost of Rs 4 and Rs 3 per unit respectively.
One unit of food A contains 2 units of vitamins , one unit of minerals and 1 calories .
One unit of food B contains 1 unit of vitamins , 2 units of minerals and 1 calories.
Find what combination of A and B should be used to have least cost but satisfying all requirements.

24. Show that a closed right circular cylinder of a given total surface area and maximum volume is such that its height is equal to the diameter of the base

25. Find the area bounded by the curve $y = 2x - x^2$ and the line $y = -x$.

26. Find the foot of the perpendicular drawn from the points A (1,0,3) to the join of points B (4,7,1) and C(3,5,3).

OR

Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3, 0, – 4).
Also find the distance between these two lines.

27.A plane P is perpendicular to a plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$and contains the line of intersection of the planes
$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0; \vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0,$ find the equation of P.

28. Using the matrix method , solve the given system of solutions :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4; \frac{4}{x} + \frac{-6}{y} + \frac{5}{z} = 1; \frac{6}{x} + \frac{9}{y} + \frac{-20}{z} = 2$$

29.

Let $A = N \times N$, N being the set of natural numbers.

Let $* : A \times A \rightarrow A$ be defined as $(a,b) * (c,d) = (ad+bc, bd)$ for all $(a,b), (c,d) \in A$.

Show that (i) '*' is commutative (ii) '*' is associative

Find the (iii) Identity element if it exists, and

(iv) The inverse element if it exists

OR

A relation R on the set of complex numbers is defined by

$$z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2}$$

Show that R is an equivalence relation.