

Mock Paper IV
MATHEMATICS
Class : XII

Time Allowed : 3 Hrs

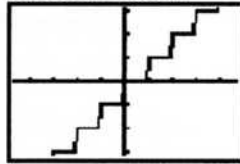
Maximum Marks: 100

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

2. This graph does not represent a function . Make the required changes in this graph, and draw the graph , so that it does represent a function.



3. For what value of λ , are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ orthogonal?
4. Find the value of ' α ' for which $\alpha(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.
5. Find the Obtuse angle of inclination to the Z-axis of a line that is inclined to X-axis at 45° and to Y-axis at 60° .
6. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts y axis.
7. Evaluate: $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx$.

8. This 3×2 matrix gives information about the number of men and women workers in three factories I, II and III who lost their jobs in the last 2 months. What do you infer from the entry in third row and second column of this matrix?

	Men workers	Women workers
Factory I	40	15

Factory II 35 40

Factory III 72 64

9. If A and B are two matrices of the same order, under what conditions is $(A-B)(A+B) = A^2 - B^2$

10. Evaluate the determinant :
$$\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$$

SECTION - B

11. Differentiate $\frac{x^3 \sqrt{5+x}}{(7-3x)^5 \sqrt[3]{8+5x}}$, w . r . t x

OR

11. If $y = a \cos(\log x) + b \sin(\log x)$, prove that $x^2 y'' + xy' + y = 0$,

12. Let $f(x) = x + 3, g(x) = x - 3; x \in N$,

Show that (i) f is not an onto function (ii) g is an onto function

13. Find the distance between the parallel planes

$$\vec{r} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 4 \text{ and } \vec{r} \cdot (6\hat{i} - 3\hat{j} + 9\hat{k}) + 13 = 0$$

14. A plane is at a distance of p units from the origin.

It makes an intercept of a, b, c with the x, y and z axis respectively. Show that it satisfies the equati

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$$

15. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards . What is the probability that

- (i) All the four cards are spades ?
- (ii) Only 3 cards are spades
- (iii) None is a spade

16. Solve the equation: $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$

17. Find the equation of tangent to the curve given by $x = a \sin^3 t, y = b \cos^3 t$ at

a point, where $t = \frac{\pi}{2}$.

18. Evaluate: $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

19. Evaluate $\int_0^1 e^{2-3x} dx$ as limit of sum

20. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbb{N}$

OR

If ω is one of the cube roots of unity, evaluate the given determinant

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

21. Show that the function f defined by $f(x) = |1-x+|x||$, $x \in \mathbb{R}$ is continuous.

OR

Show that a logarithmic function is continuous at every point in its domain.

22. Evaluate $\int \frac{(3 \sin \alpha - 2) \cos \alpha}{5 - \cos^2 \alpha - 4 \sin \alpha} d\alpha$

OR

Evaluate $\int \frac{dx}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$

Section C

23. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

.Find the points at which the function f given by $f(x) = (x - 2)^4(x + 1)^3$ has

(i) local maxima (ii) local minima (iii) point of inflexion .

Also find the (iv) local maximum value and the (v) local minimum value

24. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

25. Calculate the area (i) between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x-axis between $x = 0$ to $x = a$

(ii) Triangle AOB is in the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $OA = a$ and $OB = b$.

Find the area enclosed between the chord AB and the arc AB of the ellipse .

(iii) Find the ratio of the two areas found .

OR

Find the smaller of the two areas in which the circle $x^2 + y^2 = 2a^2$ is divided by the parabola $y^2 = ax, a > 0$

26. Find the equation of a plane that is parallel to the X-axis and passes through the line

common to two intersecting planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = -4$

27.

In a bank, principal increases continuously at the rate of 5% per year. In how many years will Rs 1000 double itself?

28. Two trainee carpenters A and B earn Rs 150 and Rs 200 per day respectively. A can make 6 frames and 4 stools per day while B can make 10 frames and 4 stools per day . How many days shall each work if it is desired to produce atleast 60 frames and 32 stools at a minimum labour cost ? Solve the problem graphically.

29. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k

Determine (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(0 < X < 3)$